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STRESS COMPUTATION ALGORITHM FOR TEMPERATURE DEPENDENT NON-LINEAR KINEMATIC HARDENING MODEL

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Abstract

In this work, we derive a stress algorithm for a non-linear kinematic hardening model. The algorithm is implemented in a FEM code. On a simple shear test, we compare the numerical results with the analytical ones.

Keywords: thermoplasticity, non-linear kinematic hardening, FEM.

1. Introduction

For the description of the Bauschinger effect, non-linear kinematic hardening is preferable, which is introduced by ARMSTRONG – FREDERICK [1] and further developed e.g. by CHABOCHE [4] and DOWELL [5].

In the finite element method, thermoelastoplastic processes are commonly studied, but there exist only few stress computation algorithms based on temperature independent non-linear hardening developed by AUFAURE [2], HARTMANN – HAUPT [6] and others. In this work, we present a stress integration algorithm for a temperature dependent, non-linear kinematic hardening model.

Thus, the paper is set out as follows. After the introduction of the constitutive relations, the boundary value problem is outlined. Then we introduce the stress computation algorithm and calculate the consistent stiffness. Last, a simple example is presented.

2. Constitutive Relations

The linearised strain tensor $\boldsymbol{\varepsilon}$ is decomposed to elastic $\boldsymbol{\varepsilon}_e$, thermal expansion $\boldsymbol{\varepsilon}_\theta$ and plastic parts $\boldsymbol{\varepsilon}_p$:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_\theta + \boldsymbol{\varepsilon}_p. \tag{1}$$

The stress $\boldsymbol{\sigma}$ is defined by an isotropic function and follows the Hooke's law

$$\boldsymbol{\sigma} = 2\mu \left[\boldsymbol{\varepsilon}_e + \frac{\nu}{1-2\nu} \operatorname{tr}(\boldsymbol{\varepsilon}_e)\, \mathbf{1} \right], \tag{2}$$

where μ denotes the shear modulus, ν is the Poisson's ratio and $\mathbf{1}$ represents the second order identity tensor. The thermal expansion is given by the relation

$$\boldsymbol{\varepsilon}_\theta = \alpha\,(\theta - \theta_0)\,\mathbf{1}, \tag{3}$$

where α means the linear thermal expansion coefficient. θ is the temperature field and θ_0 denotes the reference temperature. If we introduce the thermoelastic strain

$$\boldsymbol{\varepsilon}_{e\theta} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_\theta, \tag{4}$$

the Duhamel–Neumann law results from the *Eqs* (2)–(3):

$$\boldsymbol{\sigma} = 2\mu\left[\boldsymbol{\varepsilon}_{e\theta} + \frac{\nu}{1-2\nu}\mathrm{tr}\,(\boldsymbol{\varepsilon}_{e\theta})\,\mathbf{1}\right] - 3\kappa\alpha\,(\theta - \theta_0)\,\mathbf{1}, \tag{5}$$

where $\kappa = \frac{2\mu(1+\nu)}{3(1-2\nu)}$ denotes the bulk modulus. The thermoelastic domain is defined by a von Mises yield function F in the stress space:

$$F = \frac{1}{2}||\mathrm{dev}(\boldsymbol{\sigma} - \mathbf{x})||^2 - \frac{1}{3}k^2, \tag{6}$$

where k represents the plastic yielding parameter for isotropic hardening and depends on the accumulated inelastic strain and the temperature

$$k = k(s, \theta). \tag{7}$$

The accumulated inelastic strain s is defined in rate form as

$$\dot{s} = \sqrt{\frac{2}{3}}||\dot{\boldsymbol{\varepsilon}}_p||. \tag{8}$$

An associative flow rule is assumed

$$\dot{\boldsymbol{\varepsilon}}_p = \begin{cases} \lambda\mathbf{N} & \text{for } F = 0 \text{ and loading in the plastic range,} \\ \mathbf{0}, & \text{for all other cases,} \end{cases} \tag{9}$$

where the normal to the yield surface $\mathbf{N}$ is

$$\mathbf{N} = \frac{\mathrm{dev}(\boldsymbol{\sigma} - \mathbf{x})}{||\mathrm{dev}(\boldsymbol{\sigma} - \mathbf{x})||}. \tag{10}$$

Loading occurs in the plastic range, if

$$\dot{F}\big|_{\dot{\boldsymbol{\varepsilon}}_p = \mathbf{0}} > 0. \tag{11}$$

The proportionality factor λ is determined by the consistency condition $\dot{F} = 0$. The above system of the plastic deformation must be completed by an evolution equation for the stress type kinematic hardening tensor $\mathbf{x}$. In this work, we choose the relation as

$$\dot{\mathbf{x}} = c\dot{\boldsymbol{\varepsilon}}_p - b\dot{s}\mathbf{x} + \frac{1}{c}\frac{\partial c}{\partial \theta}\dot{\theta}\mathbf{x}, \tag{12}$$

with the temperature dependent linear and non-linear hardening parameters $c(\theta)$ and $b(\theta)$, respectively [4], [13]. This relation corresponds to the ARMSTRONG – FREDERICK [1] evolution equation of the back-stress $\dot{\mathbf{x}} = c\dot{\boldsymbol{\varepsilon}}_p - b\dot{s}\mathbf{x}$, if the material parameter does not depend on the temperature.

3. Linearization of the Principle of the Virtual Displacement

In the case of uncoupled thermoelastoplasticity, the equations for the calculation of the temperature field and the boundary value problem of the temperature dependent mechanical process can be separated.

In the coupled thermoelastoplasticity, the operator split method allows us to separate the stress and temperature computation procedures in a load step, therefore, the developed algorithm can be used in coupled thermoelastoplasticity without modifications. In this work, we do not detail the heat conduction.

The formulation of the boundary value problem starts from the principle of the virtual displacement [3]

$$\int_V \boldsymbol{\sigma} : \delta\boldsymbol{\varepsilon} \mathrm{d}V = \int_V \rho \mathbf{b}\eta \mathrm{d}V + \int_A \mathbf{t}\eta \mathrm{d}A, \tag{13}$$

where η denotes the virtual displacement field and $\delta\boldsymbol{\varepsilon} = \mathrm{sym}(\mathrm{grad}\eta)$. ρ, $\mathbf{b}$ and $\mathbf{t}$ represent the density, the body force and the surface traction, respectively. On the unknown $(n+1)$th configuration, the principle of the virtual displacement becomes

$$\Gamma\left({}^{n+1}\mathbf{u}, \eta\right) = \int_V {}^{n+1}\boldsymbol{\sigma} : \delta\boldsymbol{\varepsilon} \mathrm{d}V - \int_V {}^{n+1}\mathbf{b}\eta\rho \mathrm{d}V + \int_A {}^{n+1}\mathbf{t}\eta \mathrm{d}A, \tag{14}$$

which is a non-linear problem for the unknown displacement field ${}^{n+1}\mathbf{u}$. To solve the equation the Newton–Raphson method is applied. This requires a linearization with respect to the displacement field

$$\Gamma\left({}^{i+1}\mathbf{u}, \eta\right) = \Gamma\left({}^{i}\mathbf{u}, \eta\right) + \mathbf{D}\Gamma\left({}^{i}\mathbf{u}, \eta\right)\left[\Delta\Delta^{i}\mathbf{u}\right] = 0, \tag{15}$$

with the displacement increment $\Delta\Delta^{i}\mathbf{u} = {}^{i+1}\mathbf{u} - {}^{i}\mathbf{u}$. Here $\mathbf{D}$ denotes the Gateaux derivative. The total displacement increment is calculated as

$$\Delta^{i+1}\mathbf{u} = \sum_{j=1}^{i} \Delta\Delta^{j}\mathbf{u}. \tag{16}$$

With the introduction of the strain increments

$$\Delta\Delta^i\boldsymbol{\varepsilon} = \mathrm{sym}\left(\mathrm{grad}\left({}^{i+1}\mathbf{u} - {}^{i}\mathbf{u}\right)\right), \tag{17}$$

$$\Delta^{i+1}\boldsymbol{\varepsilon} = \sum_{j=1}^{i} \Delta\Delta^i\boldsymbol{\varepsilon}, \tag{18}$$

the Gateaux derivative of the stress ${}^i\boldsymbol{\sigma}$ can be expressed as

$$\begin{aligned}\mathbf{D}^i\boldsymbol{\sigma}\left(\Delta^i\boldsymbol{\varepsilon}\right)\left[\Delta\Delta^i\mathbf{u}\right] &= \mathbf{D}^i\boldsymbol{\sigma}\left(\Delta^i\boldsymbol{\varepsilon}\right)\left[\mathbf{D}\Delta\Delta^i\boldsymbol{\varepsilon}\left[\Delta\Delta^i\mathbf{u}\right]\right] \\ &= \frac{\mathrm{d}^i\boldsymbol{\sigma}}{\mathrm{d}\Delta^i\boldsymbol{\varepsilon}} : \Delta\Delta^i\boldsymbol{\varepsilon} = {}^i\mathbf{C}_{ep} : \Delta\Delta^i\boldsymbol{\varepsilon}.\end{aligned} \tag{19}$$

Finally, for the linearized principle of the virtual displacement yields

$$\begin{aligned}&\int_V \Delta\Delta^i\boldsymbol{\varepsilon} : {}^i\mathbf{C}_{ep} : \delta\boldsymbol{\varepsilon}\mathrm{d}V = \\ &\int_V {}^{n+1}\mathbf{b}\eta\rho\mathrm{d}V + \int_A {}^{n+1}\mathbf{t}\eta\mathrm{d}A - \int_V {}^i\boldsymbol{\sigma} : \delta\boldsymbol{\varepsilon}\mathrm{d}V.\end{aligned} \tag{20}$$

In this equation, the unknown measures are the stress ${}^i\boldsymbol{\sigma}$ and the consistent tangent operator ${}^i\mathbf{C}_{ep}$. We show the calculation of the necessary quantities in the next two sections.

4. Stress Computation Algorithm

The stress computation algorithm is based on the works by HARTMANN, LÜHRS and HAUPT [6] – [10], who developed stress computation procedures for temperature independent plasticity and viscoplasticity with non-linear kinematic hardening. In our method, the temperature dependent isotropic and kinematic hardening properties are also considered. If the material parameters do not depend on the temperature, our algorithm reduces to the one of HARTMANN [6], [7]. A return mapping algorithm is chosen, which is based on a backward-Euler step.

The initial conditions ${}^n\boldsymbol{\sigma}$, ${}^n\mathbf{x}$, ${}^ns$ and ${}^n\theta$ are known from the last equilibrium state. We will calculate the measures ${}^i\boldsymbol{\sigma}$, ${}^i\mathbf{x}$ and ${}^is$ at the updated state from the strain increment $\Delta^i\boldsymbol{\varepsilon}$ and from the temperature ${}^{n+1}\theta$.

First, the thermoelastic predictor, the trial stress ${}^t\boldsymbol{\sigma}$ is obtained from the strain and temperature increments

$${}^t\boldsymbol{\sigma} = {}^n\boldsymbol{\sigma} + \mathbf{C}_e : \Delta\boldsymbol{\varepsilon} - 3\kappa\alpha\Delta\theta\mathbf{1}, \tag{21}$$

where $\Delta\theta = {}^{n+1}\theta - {}^{n}\theta$. Then the trial back-stress is evaluated

$$ {}^{t}\mathbf{x} = \left[1 - \frac{\Delta\theta}{c\left({}^{n+1}\theta\right)} \left.\frac{\partial c}{\partial\theta}\right|_{{}^{n+1}\theta}\right]^{-1} {}^{n}\mathbf{x}. \tag{22} $$

The next step is to check the yield condition. In this step, if

$$ \frac{1}{2}||\mathrm{dev}\left({}^{t}\boldsymbol{\sigma}\right) - {}^{t}\mathbf{x}|| - \frac{1}{3}k\left({}^{n}s, {}^{n+1}\theta\right) < 0, \tag{23} $$

then the deformation is thermoelastic and the variables of the ith state are simply updated as

$$ {}^{i}\boldsymbol{\sigma} = {}^{t}\boldsymbol{\sigma}, \qquad {}^{i}\mathbf{x} = {}^{t}\mathbf{x} \qquad {}^{i}s = {}^{n}s. \tag{24} $$

Otherwise, the plastic corrector is applied as follows. The flow rule is approximated by

$$ \Delta^{i}\boldsymbol{\varepsilon}_{i} = \Delta^{i}\lambda^{i}\mathbf{N}, \tag{25} $$

where the normal to the yield surface is calculated as

$$ {}^{i}\mathbf{N} = \frac{\mathrm{dev}\left({}^{i}\boldsymbol{\sigma}\right) -{}^{i}\,\mathbf{x}}{||\mathrm{dev}\left({}^{i}\boldsymbol{\sigma}\right) -{}^{i}\,\mathbf{x}||}. \tag{26} $$

With *Eq.* (25), the elasticity relation, the back-stress and the accumulated inelastic strain are given as

$$ {}^{i}\boldsymbol{\sigma} = {}^{n}\boldsymbol{\sigma} + \mathbf{C}_{e} : \left[\Delta\boldsymbol{\varepsilon} - \Delta^{i}\boldsymbol{\varepsilon}_{i}\right] - 3\kappa\alpha\Delta\theta\mathbf{1} = {}^{t}\boldsymbol{\sigma} - 2\mu\Delta^{i}\lambda^{i}\mathbf{N}, \tag{27} $$

$$ \begin{aligned} {}^{i}\mathbf{x} = {}^{n}\mathbf{x} + c\left({}^{n+1}\theta\right)\Delta^{i}\lambda^{i}\mathbf{N} - b\left({}^{n+1}\theta\right)\sqrt{\frac{2}{3}}\Delta^{i}\lambda^{i}\mathbf{x} \\ + \frac{\Delta^{i}\theta}{c\left({}^{n+1}\theta\right)} \left.\frac{\partial c}{\partial\theta}\right|_{{}^{n+1}\theta}^{i}\mathbf{x}, \end{aligned} \tag{28} $$

$$ {}^{i}s = {}^{n}s + \sqrt{\frac{2}{3}}\Delta^{i}\lambda. \tag{29} $$

It is convenient to write the yield condition in the form

$$ ||\mathrm{dev}\left({}^{i}\boldsymbol{\sigma}\right) - {}^{i}\mathbf{x}|| - \sqrt{\frac{2}{3}}k\left({}^{i}s, {}^{n+1}\theta\right) = 0. \tag{30} $$

Now, we solve *Eqs* (25) – (30). The back-stress tensor ${}^{i}\mathbf{x}$ is expressed from *Eq.* (27):

$$ {}^{i}\mathbf{x} = {}^{i}\beta\left({}^{n}\mathbf{x} + c\left({}^{n+1}\theta\right)\Delta^{i}\lambda^{i}\mathbf{N}\right), \tag{31} $$

with

$$ {}^{i}\beta = \left[1 + b\left({}^{n+1}\theta\right)\sqrt{\frac{2}{3}}\Delta^{i}\lambda - \frac{\Delta\theta}{c\left({}^{n+1}\theta\right)} \left.\frac{\partial c}{\partial\theta}\right|_{{}^{n+1}\theta}\right]^{-1}. \tag{32} $$

Following an idea of SIMO – TAYLOR [12], the difference tensor between the deviatoric part of the elasticity relation (27) and the back-stress tensor (31) is derived. Using *Eq.* (25) yields

$$\operatorname{dev}\left({}^{i}\boldsymbol{\sigma}\right) - {}^{i}\mathbf{x} = \operatorname{dev}\left({}^{t}\boldsymbol{\sigma}\right) - {}^{i}\beta^{n}\mathbf{x} - \frac{2\mu + {}^{i}\beta c\left({}^{n+1}\theta\right)}{||\operatorname{dev}\left({}^{i}\boldsymbol{\sigma}\right) - {}^{i}\mathbf{x}||}\left(\operatorname{dev}\left({}^{i}\boldsymbol{\sigma}\right) - {}^{i}\mathbf{x}\right). \tag{33}$$

After rearranging the expression results

$$\operatorname{dev}\left({}^{i}\boldsymbol{\sigma}\right) - {}^{i}\mathbf{x} = \frac{\operatorname{dev}\left({}^{t}\boldsymbol{\sigma}\right) - {}^{i}\beta^{n}\mathbf{x}}{1 + \Delta^{i}\lambda\dfrac{2\mu + {}^{i}\beta c\left({}^{n+1}\theta\right)}{||\operatorname{dev}\left({}^{i}\boldsymbol{\sigma}\right) - {}^{i}\mathbf{x}||}}. \tag{34}$$

We introduce for the norm in the above equation the function

$${}^{i}\gamma = ||\operatorname{dev}\left({}^{i}\boldsymbol{\sigma}\right) - {}^{i}\mathbf{x}|| = \sqrt{\frac{2}{3}}k\left({}^{i}s, {}^{n+1}\theta\right), \tag{35}$$

furthermore

$${}^{i}\omega = 1 + \Delta^{i}\lambda\frac{2\mu + {}^{i}\beta c\left({}^{n+1}\theta\right)}{{}^{i}\gamma}. \tag{36}$$

With the definition

$${}^{i}\Xi = \operatorname{dev}\left({}^{t}\boldsymbol{\sigma}\right) - {}^{i}\beta^{n}\mathbf{x}, \tag{37}$$

Eq. (34) can be rewritten as

$$\operatorname{dev}\left({}^{i}\boldsymbol{\sigma}\right) - {}^{i}\mathbf{x} = \frac{1}{{}^{i}\omega}{}^{i}\Xi. \tag{38}$$

Finally, the yield condition (30) becomes

$$\phi = \frac{1}{{}^{i}\omega}||{}^{i}\Xi|| - {}^{i}\gamma = 0. \tag{39}$$

This remaining equation represents one non-linear scalar equation to calculate the unknown plastic multipliers $\Delta^{i}\lambda$. To solve the equation a numerical method is necessary, a local Newton–Raphson procedure was applied.

The stress computation algorithm is summarized as follows:

1. Given: ${}^{n}\boldsymbol{\sigma}, \Delta^{i}\boldsymbol{\varepsilon}, {}^{n}\mathbf{x}, {}^{n}s, {}^{n}\theta, {}^{n+1}\theta \;\rightarrow\; \Delta\theta = {}^{n+1}\theta - {}^{n}\theta$
2. Thermoelastic predictor:
   ${}^{t}\boldsymbol{\sigma} = {}^{n}\boldsymbol{\sigma} + \mathbf{C}_{e} : \Delta^{i}\boldsymbol{\varepsilon} - 3\kappa\alpha\Delta\theta\mathbf{1}$
3. Calculate the trial back-stress:
   ${}^{t}\mathbf{x} = \left[1 - \frac{\Delta\theta}{c\left({}^{n+1}\theta\right)}\left.\frac{\partial c}{\partial\theta}\right|_{{}^{n+1}\theta}\right]^{-1}{}^{n}\mathbf{x}$

4. Check the yield condition:
   If $||{}^t\boldsymbol{\sigma} - {}^t\mathbf{x}|| - \sqrt{\frac{2}{3}}k\left({}^ns, {}^{n+1}\theta\right) < 0$
   then: ${}^i\boldsymbol{\sigma} = {}^t\boldsymbol{\sigma}$, ${}^i\mathbf{x} = {}^t\mathbf{x}$, ${}^is = {}^ns$
   exit
5. otherwise:
   Find $\Delta^i\lambda$ by local iteration, that is solve $\phi\left(\Delta^i\lambda\right) = 0$ for $\Delta^i\lambda$.
6. Calculate the variables
   ${}^i\mathbf{N} = \frac{1}{{}^i\gamma^i\omega}{}^i\Xi, \qquad {}^i\boldsymbol{\sigma} = {}^t\boldsymbol{\sigma} - 2\mu\Delta^i\lambda^i\mathbf{N}$
   ${}^is = {}^ns + \sqrt{\frac{2}{3}}\Delta^i\lambda \; {}^i\mathbf{x} = {}^i\beta\left({}^n\mathbf{x} + c\left({}^{n+1}\theta\right)\Delta^i\lambda^i\mathbf{N}\right)$
   exit

5. Calculation of the Consistent Stiffness

In the FEM calculations, the quadratic convergence rate needs the consistent linearization of the constitutive relation. The consistent tangent operator is the derivative of the stress, coming out from the stress calculation algorithm, with respect to the current strain

$$ {}^i\mathbf{C}_{ep} = \frac{\mathrm{d}^i\boldsymbol{\sigma}}{\mathrm{d}^i\boldsymbol{\varepsilon}} = \frac{\mathrm{d}^i\boldsymbol{\sigma}}{\mathrm{d}\Delta^i\boldsymbol{\varepsilon}}. \tag{40}$$

After some calculations, the consistent stiffness leads to

$$ {}^i\mathbf{C}_{ep} = \kappa\mathbf{1}\otimes\mathbf{1} + \delta_1\left[\mathbf{I} - \frac{1}{3}\mathbf{1}\otimes\mathbf{1}\right] - \delta_2{}^i\mathbf{N}\otimes{}^i\mathbf{N} - \delta_3{}^i\mathbf{x}\otimes{}^i\mathbf{N} \tag{41}$$

with

$$\delta_1 = 2\mu\left[1 - \frac{2\mu\Delta^i\lambda}{{}^i\omega^i\gamma}\right], \tag{42}$$

$$\delta_2 = \frac{4\mu^2}{{}^i\omega}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\Delta^i\lambda}\right)^{-1}\left[-1 + \Delta^i\lambda\left(\frac{1}{{}^i\omega}\frac{\mathrm{d}^i\omega}{\mathrm{d}\Delta^i\lambda} + \frac{1}{{}^i\gamma}\frac{\mathrm{d}^i\gamma}{\mathrm{d}\Delta^i\lambda}\right)\right], \tag{43}$$

$$\delta_3 = \frac{4\mu^2\Delta^i\lambda}{{}^i\omega^{2i}\gamma}\frac{\mathrm{d}\Delta^i\beta}{\mathrm{d}^i\lambda}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\Delta^i\lambda}\right)^{-1}. \tag{44}$$

Because of the last term in (41), the resulting tangential stiffness matrix is non-symmetric. The non-symmetry results from the non-linear kinematic hardening and from the temperature dependent linear kinematic hardening variable c. The tangent operator becomes symmetric in the following cases:

– the plastic multiplier tends to zero $\left(\Delta^i\lambda \to 0\right)$,
– the non-linear kinematic hardening parameter $b = 0$ and the linear kinematic hardening parameter c does not depend on the temperature $\left(\frac{\mathrm{d}c}{\mathrm{d}\theta} = 0\right)$,
– for radial processes (i.e. ${}^i\mathbf{N}$ and ${}^i\mathbf{x}$ are parallel).

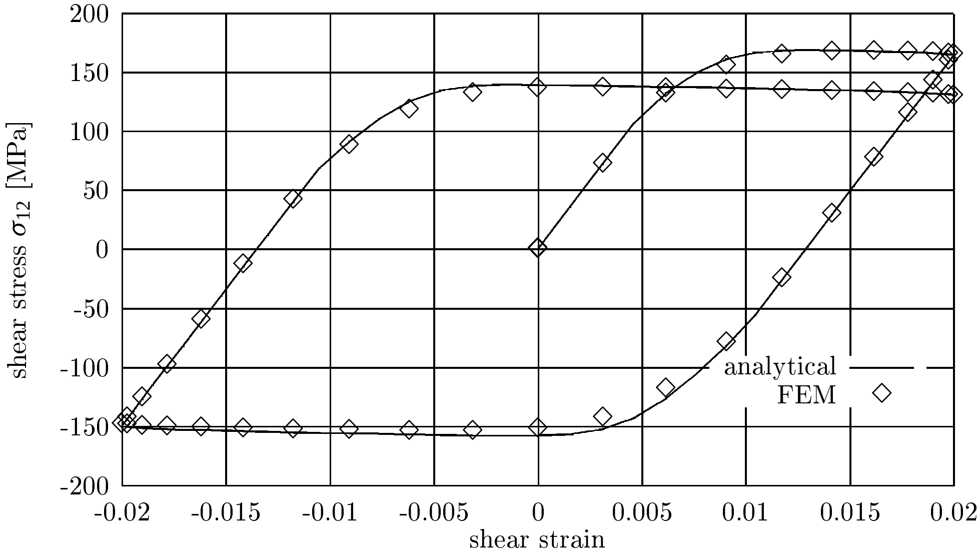

Fig. 1. Shear stress versus shear strain

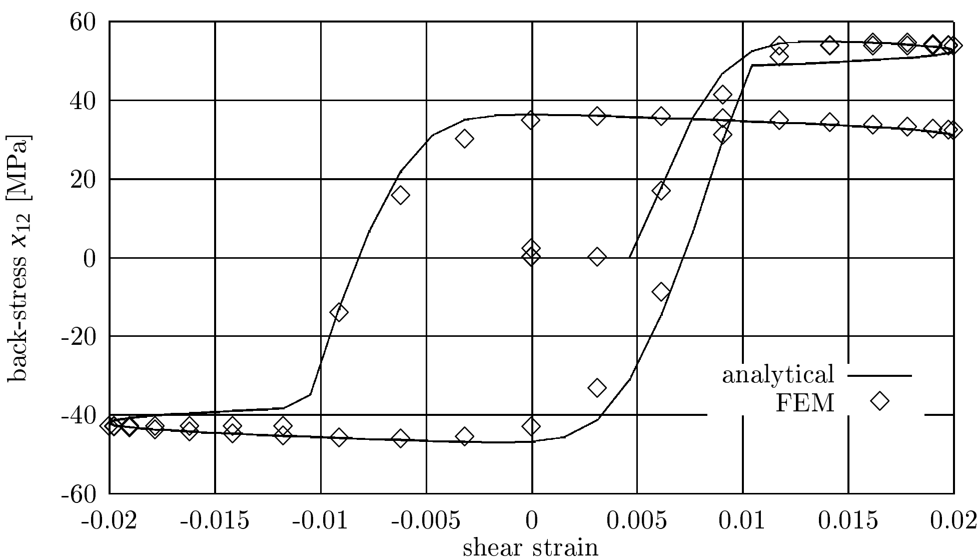

Fig. 2. Back-stress versus shear strain

6. Example

The stress algorithm was implemented in the FEM System MARC [11] as the user subroutine `hypela2.f`. We compared the numerical results with the analytical ones. In the numerical example, we show the reliability of our algorithm on a simple shear test for one cycle. For this simple problem, the analytical solution can easily be derived.

The material parameters are taken as

$$\begin{array}{lll} \mu = 23077 \text{ MPa}, & \nu = 0.3, & \alpha = 0, \\ c = 100000 - 500\theta \text{ MPa}, & b = 1500 - \theta, & k = 200 - 0.3\theta \text{ MPa}. \end{array} \tag{45}$$

The calculations were performed with displacement control, where the time dependent functions of the shear strain γ and the temperature θ are

$$\gamma = 0.02 \sin(2.5\pi t), \qquad \theta = 100t^{\circ}\text{C}, \tag{46}$$

where in the quasi-static analysis the time parameter t varies between 0 and 1. The load was applied in 50 uniform time steps.

Figs 1 and *2* show the non-vanishing parts of the stress $\boldsymbol{\sigma}_{12}$ and the back-stress x_{12}, respectively, and compare the analytical and numerical solutions.

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